00 P1 B7	**3C** •[1] $\overrightarrow{VK} = \overrightarrow{VA} + \overrightarrow{AB} + \overrightarrow{BK}$ *or* $\overrightarrow{VK} = \overrightarrow{VB} + \overrightarrow{BK}$ •[2] $\overrightarrow{BK} = \frac{1}{4}\overrightarrow{BC}$ $(or$ $\frac{1}{4}\overrightarrow{AD})$ *or* $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ *or* $\begin{pmatrix} -1 \\ -7 \\ -17 \end{pmatrix}$ •[3] $\overrightarrow{VK} = \begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$
00 P2 B7	**2C** •[1] $u.v = 2t - 20 + 3t$ •[2] $u.v = 0 \Rightarrow t = 4$
00 P2 B9	**2C, 6C** •[1] $B = (3, 2, 15)$ •[2] $\cos A\hat{B}C = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{BA \times BC}$ •[3] $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$ •[4] $\overrightarrow{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$ •[5] $BA = \sqrt{107}$, $BC = \sqrt{249}$ •[6] $\overrightarrow{BA}.\overrightarrow{BC} = -7$ •[7] $A\hat{B}C = 92.5°$
01 P1 Q3	**3C, 3C** •[1] *e.g.* $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$ •[2] *e.g.* $\overrightarrow{BC} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix} = \frac{4}{3}\overrightarrow{AB}$ •[3] a) a common direction exists *and* b) a common point exists so A, B, C collinear ans: the road ABC is straight

	$\bullet^4$ $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$ $\bullet^5$ $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9$ $\bullet^6$ $= 0$ so AB is at right angles to BD
01 P2 Q4	**7C** $\bullet^1$ *use* $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ stated $\bullet^2$ $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$ $\bullet^3$ $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$ $\bullet^4$ $\lvert\overrightarrow{BA}\rvert = \sqrt{62}$ $\bullet^5$ $\lvert\overrightarrow{BC}\rvert = \sqrt{52}$ $\bullet^6$ $\overrightarrow{BA}.\overrightarrow{BC} = 18$ $\bullet^7$ $A\hat{B}C = 71.5$
02 P1 Q2	**3C** $\bullet^1$ $\overrightarrow{PR} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ $\bullet^2$ $\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{PR}$ $\bullet^3$ $Q = (3, 1, -2)$
02 P2 Q2	**1C, 2C, 4C**

	$\bullet^1$ $B=(6,6,0)$ $\bullet^2$ $\vec{DA}=\begin{pmatrix}3\\-3\\-8\end{pmatrix}$ $\bullet^3$ $\vec{DB}=\begin{pmatrix}3\\3\\-8\end{pmatrix}$ $\bullet^4$ $\cos A\hat{D}B=\dfrac{\vec{DA}.\vec{DB}}{\lvert\vec{DA}\rvert\lvert\vec{DB}\rvert}$ $\bullet^5$ $\lvert\vec{DA}\rvert=\sqrt{82}\,,\quad \lvert\vec{DB}\rvert=\sqrt{82}$ $\bullet^6$ $\vec{DA}.\vec{DB}=64$ $\bullet^7$ $A\hat{D}B=38.7°$
6.(JAN) 02 P1	**2C, 2C** $\bullet^1$ $u+3v=\begin{pmatrix}4\\1\\1\end{pmatrix}$ $\bullet^2$ $u-3v=\begin{pmatrix}-2\\13\\-5\end{pmatrix}$ $\bullet^3$ $\begin{pmatrix}4\\1\\1\end{pmatrix}.\begin{pmatrix}-2\\13\\-5\end{pmatrix}$ $\bullet^4$ $=-8+13-5=0$
7.(JAN) 02 P1	**3A** $\bullet^1$ $a.b+a.c$ $\bullet^2$ $\lvert a\rvert\lvert b\rvert\cos 60°+\lvert a\rvert\lvert c\rvert\cos 120°$ $\bullet^3$ $2\times2\times\frac{1}{2}+2\times2\times-\frac{1}{2}=0$ **OR** $\bullet^1$ $\vec{PQ}.\vec{QS}$ $\bullet^2$ $Q\hat{R}S=120°\Rightarrow R\hat{Q}S=30°\Rightarrow P\hat{Q}S=90°$ $\bullet^3$ $\vec{PQ}.\vec{QS}=\lvert PQ\rvert\lvert QS\rvert\cos 90°=0$
8.(JAN) 02 P2	**4C** $\bullet^1$ $\vec{AB}=\begin{pmatrix}6\\-9\\-12\end{pmatrix}$ *or* $\vec{BC}=\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ *or* $\vec{AC}=\begin{pmatrix}8\\-12\\-16\end{pmatrix}$ $\bullet^2$ 2nd vector *and e.g.* $\vec{AB}=3\vec{BC}$ $\left(or\ \begin{pmatrix}6\\-9\\-12\end{pmatrix}=3\begin{pmatrix}2\\-3\\-4\end{pmatrix}\right)$ $\bullet^3$ *e.g.* $\vec{AB},\vec{BC}$ have common direction, B common pt., so A, B, C collinear $\bullet^4$ AB:BC = 3:1
03 P1 Q3	**2C** $\bullet^1$ for perpendicularity $u.v=0$ $\bullet^2$ $\begin{pmatrix}3\\2\\0\end{pmatrix}.\begin{pmatrix}2\\-3\\4\end{pmatrix}=6-6+0=0$ ans : vectors are perpendicular
03 P1	**3C**

Q6	$\bullet^1$ $\vec{AB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ $\bullet^2$ $\vec{AD} = 3\,\vec{AB} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$ $\bullet^3$ $D = (8, 3, -1)$
03 P2 Q9	**4B** $\bullet^1$ $a.(a+b) = a.a + a.b$ $\bullet^2$ $a.b = 5 \times 4\cos(\theta)$ $\bullet^3$ $a.a \ = 5^2$ $\bullet^4$ $[\cos(\theta) = 0.55] \ \Rightarrow \ \theta = 56.6^\circ$ 0.99 radians
04 P1 Q5	$\bullet^1$ $\vec{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\bullet^2$ $\vec{AC} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} = \frac{3}{2} \times \vec{AB}$ $\bullet^1$ $\vec{BC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\bullet^1$ $\vec{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\bullet^2$ $\vec{AC} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix} = 3 \times \vec{BC}$ $\bullet^2$ $\vec{BC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \frac{1}{2} \times \vec{AB}$ **OR** $\bullet^3$ $\vec{AB}$ & $\vec{AC}$ have <u>common</u> <u>direction</u> and <u>common point</u> Hence **A**,B and C collinear $\bullet^4$ $\vec{AD} = \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix}$ $\bullet^5$ $D = (5, 20, -9)$
04 P2 Q2	**C, C**

	•[1] $\vec{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ •[2] $\vec{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$ •[3] $\cos PQR = \dfrac{\vec{QP}.\vec{QR}}{\lvert\vec{QP}\rvert \times \lvert\vec{QR}\rvert}$ stated •[4] $\vec{QP}.\vec{QR} = 6$ •[5] $\lvert\vec{QP}\rvert = \sqrt{14}$ •[6] $\lvert\vec{QR}\rvert = \sqrt{27}$ •[7] $P\hat{Q}\boldsymbol{R} = 72.0°$
05 P1 Q3	**C, C** •[1] $\overrightarrow{DB} = \begin{pmatrix} 12-6 \\ 6-3 \\ 0-9 \end{pmatrix}$ •[2] $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DB}$ •[3] $\overrightarrow{DF} = \frac{2}{3}\begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$ •[4] $\mathrm{D} = (6,3,9)\ so\ \mathrm{F} = (10,5,3)$ •[5] $\overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$ **OR**

	• [1] $\overrightarrow{DF} = 2\overrightarrow{FB}$ s/i by •2 • [2] $\boldsymbol{f} - \boldsymbol{d} = 2\boldsymbol{b} - 2\boldsymbol{f}$ • [3] $3\boldsymbol{f} = 2\begin{pmatrix}12\\6\\0\end{pmatrix} + \begin{pmatrix}6\\3\\9\end{pmatrix}$ • [4] $\mathrm{F} = (10,5,3)$ *[Note 1]*	**OR** • [1] $\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$ • [2] $\overrightarrow{AF} = \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BD}$ • [3] $\overrightarrow{AF} = \begin{pmatrix}0\\6\\0\end{pmatrix} + \frac{1}{3}\left[\begin{pmatrix}6\\3\\9\end{pmatrix} - \begin{pmatrix}12\\6\\0\end{pmatrix}\right]$ • [4] $\overrightarrow{AF} = \begin{pmatrix}-2\\5\\3\end{pmatrix}$ • [5] $(\mathrm{A} = (12,0,0 \text{ so})\ \mathrm{F} = (10,5,3)$
05 P2 Q4	**C, C** • [1] $\vec{TA} = \begin{pmatrix}-5\\15\\1\end{pmatrix}$ • [2] $\vec{TB} = \begin{pmatrix}-40\\15\\2\end{pmatrix}$ • [3] $\lvert \vec{TA} \rvert = \sqrt{251}$ • [4] $\lvert \vec{TB} \rvert = \sqrt{1829}$ • [5] $\vec{TA}.\vec{TB} = 427$ • [6] $\cos(A\hat{T}B) = \dfrac{\vec{TA}.\vec{TB}}{\lvert\vec{TA}\rvert\lvert\vec{TB}\rvert}$ • [7] $A\hat{T}B = 50\cdot 9^\circ \ OR \ 0.889^c$ OR 56.6 grads	
05 P2 Q10	**A** • [1] $\boldsymbol{a.a} + \boldsymbol{a.b} + \boldsymbol{a.c}$ see CAVE • [2] $\boldsymbol{a.a} = 9$ • [3] $\boldsymbol{a.c} = \frac{9}{2}$ • [4] $\boldsymbol{a.b} = 0$ *and a total of* $13\frac{1}{2}$	

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CAVE

$a.(a+b+c) = a.a + a.b + a.c$
followed by
$a.a = 9$
earns $\bullet 1$ *and* $\bullet 2$.

but

$a.(a+b+c) = a.a + a.b + a.c$
followed by
$a.a = 9,\ a.c = 9,\ a.b = 6$
earns $\bullet 1$ *only*.

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<tr><td>06 P1
Q9</td><td>

C, C, C, C

$\bullet^1$ $\boldsymbol{u}.\boldsymbol{v} = k^3.1 + 1.\left(3k^2\right) + \left(k+2\right).(-1)$ stated or implied by •2 before completion

$\bullet^2$ $k^3 + 3k^2 - k - 2 = 1$ *and complete* **2 marks**

$\bullet^3$ know to use $k = -3$

$\bullet^4$ $-27 + 27 - (-3) - 3 = 0 \Rightarrow x + 3$ *is a factor*

$\bullet^5$ $(k+3)(k^2\ \ldots.)$

$\bullet^6$ $(k+3)\left(k^2 - 1\right)$

$\bullet^7$ $(k+3)(k+1)(k-1)$ stated explicitly **5 marks**

$\bullet^8$ $k = 1$ **1 mark**

$\bullet^9$ $\boldsymbol{u} = \begin{pmatrix}1\\1\\3\end{pmatrix}, \boldsymbol{v} = \begin{pmatrix}1\\3\\-1\end{pmatrix}$ stated or implied by •10

$\bullet^{10}$ $\lvert \boldsymbol{u} \rvert = \sqrt{11}$ *and* $\lvert \boldsymbol{v} \rvert = \sqrt{11}$

$\bullet^{11}$ $\cos\theta = \dfrac{1}{11}$ **3 marks**

N.B.
•9 and •10 may be cross-marked.

OR

$\bullet^3$ $\begin{array}{r|l} -3 & \\ \hline & \end{array}$

$\bullet^4$ $\begin{array}{r|rrrr} -3 & 1 & 3 & -1 & -3 \\ & & -3 & 0 & 3 \\ \hline & 1 & 0 & -1 & 0 \end{array}$

$\bullet^5$ $"f(-3)" = 0$ *so* $(k+3)$ *is a factor*

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<tr><td>06 P2
Q6</td><td>

C, C, B

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	$\bullet^1$ $\overrightarrow{\mathrm{PQ}}=\begin{pmatrix}4\\0\\-3\end{pmatrix}$ $\bullet^2$ $\lvert\overrightarrow{\mathrm{PQ}}\rvert=5$ $\bullet^3$ $\begin{pmatrix}\frac{4}{5}\\0\\-\frac{3}{5}\end{pmatrix}$ Accept $\frac{1}{5}\begin{pmatrix}4\\0\\-3\end{pmatrix}$ for $\bullet^3$.
07 P1 Q2	**C** $\bullet^1$ $AB=\boldsymbol{b}-\boldsymbol{a}$ stated or implied by $\bullet^2$ $\bullet^2$ $\overrightarrow{AB}=\begin{pmatrix}3\\2\\3\end{pmatrix}$ $\bullet^3$ $\overrightarrow{BC}=\begin{pmatrix}6\\4\\6\end{pmatrix}$ $\bullet^4$ $C=(7,7,8)$ **OR** $\bullet^1$ $c-b=2b-2a$ $\bullet^2$ $c=3b-2a$ $\bullet^3$ $c=3\begin{pmatrix}1\\3\\2\end{pmatrix}-2\begin{pmatrix}-2\\1\\-1\end{pmatrix}$ $\bullet^4$ $C=(7,7,8)$
07 P2 Q1	**C, C, C** $\bullet^1$ $G=(0,2,2)$ $\bullet^2$ $\boldsymbol{p}=\begin{pmatrix}0\\1\\1\end{pmatrix}$ $\bullet^3$ $\boldsymbol{q}=\begin{pmatrix}1\\2\\1\end{pmatrix}$ $\boldsymbol{p}$ and $\boldsymbol{q}$ must be stated explicitly as a column (or row) vector $\bullet^4$ $\cos \mathrm{P\hat{O}Q}=\frac{\boldsymbol{p}.\boldsymbol{q}}{\lvert\boldsymbol{p}\rvert\lvert\boldsymbol{q}\rvert}$ stated or implied (s/i) by $\bullet^8$ $\bullet^5$ $\lvert\boldsymbol{p}\rvert=\sqrt{2}$ $\bullet^6$ $\lvert\boldsymbol{q}\rvert=\sqrt{6}$ $\bullet^7$ $\boldsymbol{p}.\boldsymbol{q}=3$ $\bullet^8$ $\mathrm{P\hat{O}Q}=30^\circ$ $[\mathit{radians}: \frac{\pi}{6}\ (0.524);\ \mathit{gradians}: 33.3\ ]$
08 P2 Q2	**C, C, C**

$\bullet^1$ $P = (8, 0, 4)$

$\bullet^2$ $Q = (0, 4, 3)$

$\bullet^3$ $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$

$\bullet^4$ $\overrightarrow{PA} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$

$\bullet^5$ $\cos QPA = \dfrac{\overrightarrow{PQ}.\overrightarrow{PA}}{|\overrightarrow{PQ}||\overrightarrow{PA}|}$ *stated / implied by* $\bullet^9$

$\bullet^6$ $\overrightarrow{PQ}.\overrightarrow{PA} = 4$

$\bullet^7$ $|\overrightarrow{PQ}| = \sqrt{81}$

$\bullet^8$ $|\overrightarrow{PA}| = \sqrt{16}$

$\bullet^9$ $83{\cdot}6°$, 1.459 *radians*, 92.9 *gradians*

---

09 P1 Q22

C, C

$\bullet^1$ $\overrightarrow{DE} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ ***or*** $\overrightarrow{EF} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

$\bullet^2$ 2nd column vector ***and*** $\overrightarrow{DE} = 3\overrightarrow{EF}$

$\bullet^3$ $\overrightarrow{DE}$ and $\overrightarrow{EF}$ have common point and common direction
hence D, E and F collinear

$\bullet^4$ $3:1$ stated explicitly

$\bullet^5$ $\overrightarrow{GE} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$

$\bullet^6$ $\overrightarrow{DE}.\overrightarrow{GE} = 0$

$\bullet^7$ $-9(1-k) + 6 \times (-3) + 12 \times (-3)$

$\bullet^8$ $k = 7$

Options for $\bullet^1$ *to* $\bullet^3$ **:**

1

$\bullet^1$ $\overrightarrow{DE} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ $\bullet^2$ $\overrightarrow{DF} = \begin{pmatrix} -12 \\ 8 \\ 16 \end{pmatrix} = \frac{4}{3}\overrightarrow{DE}$

$\bullet^3$ $\overrightarrow{DE}$ and $\overrightarrow{DF}$ have common point and common direction
hence D, E and F collinear

2

$\bullet^1$ $\overrightarrow{EF} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$ $\bullet^2$ $\overrightarrow{DF} = \begin{pmatrix} -12 \\ 8 \\ 16 \end{pmatrix} = 4\overrightarrow{EF}$

$\bullet^3$ $\overrightarrow{EF}$ and $\overrightarrow{DF}$ have common point and common direction
hence D, E and F collinear

Notes

$\bullet^3$ can **only** be awarded if a candidate has stated
* "common point",
* "common direction" (or "parallel")
* and "collinear"

The "=0" shown at $\bullet^6$ must appear somewhere before $\bullet^8$.

	If $\boldsymbol{a}$ and $\boldsymbol{b}$ are not defined, then merely quoting $\boldsymbol{a.b}=0$ does not gain $\bullet^{6}$.	
09 P2 Q7	**B, A/B** $\bullet^{1}$ $\boldsymbol{p.q}+\boldsymbol{p.r}$ — **s / l by ($\bullet^{2}$ and $\bullet^{4}$)** $\bullet^{2}$ $4\times3\cos30°$ — **s / l by $\bullet^{3}$** $\bullet^{3}$ $6\sqrt{3}$ (10.4) $\bullet^{4}$ $\boldsymbol{p.r}=0$ — **explicitly stated** $\bullet^{5}$ $-\|\boldsymbol{r}\|\times3\cos120°$ $\bullet^{6}$ $r=\frac{3}{2}$ *and* ... $\frac{9}{4}$ $\bullet^{7}$ $\boldsymbol{q}+\boldsymbol{r}\equiv$ from D to the projection of A onto DC $\bullet^{8}$ $\|\boldsymbol{q}+\boldsymbol{r}\|=\frac{3\sqrt{3}}{2}$ $\bullet^{9}$ $\boldsymbol{p}-\boldsymbol{q}\equiv\overrightarrow{AC}$ $\bullet^{10}$ $\|\boldsymbol{p}-\boldsymbol{q}\|=\sqrt{\left(4-\frac{3\sqrt{3}}{2}\right)^2+\left(\frac{3}{2}\right)^2}$ (2.05) **Alternatives 1** 1 For $\bullet^{7}$ and $\bullet^{8}$: $\bullet^{7}$ √ $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})=\|\boldsymbol{p}\|\ \|\boldsymbol{q}+\boldsymbol{r}\|\cos0$ $6\sqrt{3}\ =4\|\boldsymbol{q}+\boldsymbol{r}\|\times1$ $\bullet^{8}$ √ $\|\boldsymbol{q}+\boldsymbol{r}\|=\frac{6\sqrt{3}}{4}=\frac{3\sqrt{3}}{2}$ 2 For $\bullet^{9}$, $\bullet^{10}$: Using right-angled Δ ABC $\bullet^{9}$ $\overrightarrow{AC}=\boldsymbol{p}-\boldsymbol{q}$, *and* $\|\overrightarrow{AB}\|=4-\frac{3\sqrt{3}}{2}$, $\|\overrightarrow{BC}\|=\frac{3}{2}$ *and* $A\hat{C}B=43.06°$ $\bullet^{10}$ *use* $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})=\frac{9}{4}$ *to get* $\|\boldsymbol{p}-\boldsymbol{q}\|=2.05$ **Alternatives 2** 3 For $\bullet^{7}$, $\bullet^{8}$, $\bullet^{9}$, $\bullet^{10}$: Set up a coord system with origin at D $\bullet^{7}$ $C=(4,0), A=\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right), B=\left(4,\frac{3}{2}\right)$ $\bullet^{8}$ $\boldsymbol{p}=\begin{pmatrix}4\\0\end{pmatrix}, \boldsymbol{q}=\begin{pmatrix}\frac{3\sqrt{3}}{2}\\\frac{3}{2}\end{pmatrix}, \boldsymbol{r}=\begin{pmatrix}0\\-\frac{3}{2}\end{pmatrix}$ $\bullet^{9}$ $\boldsymbol{q}+\boldsymbol{r}=\begin{pmatrix}\frac{3\sqrt{3}}{2}\\0\end{pmatrix}$ *and* $\|\boldsymbol{q}+\boldsymbol{r}\|=2.60$ $\bullet^{10}$ $\boldsymbol{p}-\boldsymbol{q}=\begin{pmatrix}4-\frac{3\sqrt{3}}{2}\\-\frac{3}{2}\end{pmatrix}$ *and* $\|\boldsymbol{p}-\boldsymbol{q}\|=2.05$	
10 P2 Q1	interpret midpoint for M interpret ratio for N intepret diagram process vectors	$\bullet^{1}$ (0, 1, 0) $\bullet^{2}$ (4, 2, 2) $\bullet^{3}$ $\overrightarrow{VM}=\begin{pmatrix}0\\-1\\-3\end{pmatrix}$ $\bullet^{4}$ $\overrightarrow{VN}=\begin{pmatrix}4\\0\\-1\end{pmatrix}$

	know to use scalar product	•5 $\cos M\hat{V}N = \dfrac{\overrightarrow{VM}\,.\,\overrightarrow{VN}}{\lvert\overrightarrow{VM}\rvert\lvert\overrightarrow{VN}\rvert}$
	find scalar product	•6 $\overrightarrow{VM}\,.\,\overrightarrow{VN} = 3$
	find magnitude of a vector	•7 $\lvert\overrightarrow{VM}\rvert = \sqrt{10}$
	find magnitude of a vector	•8 $\lvert\overrightarrow{VN}\rvert = \sqrt{17}$
	evaluate angle	•9 $76{\cdot}7^\circ$ or $1{\cdot}339$ rads or $85{\cdot}2$ grads
11 P2 Q1	state coordinates of B	•1 $(4,\ 4,\ 0)$
	state components of $\overrightarrow{DB}$	•2 $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$
	state coordinates of M	•3 $(2,\ 0,\ 0)$ **stated, or implied**
	state components of $\overrightarrow{DM}$	•4 $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$
	know to use scalar product	•5 $\cos B\hat{D}M = \dfrac{\overrightarrow{DB}\,.\,\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$
	find scalar product	•6 $\overrightarrow{DB}\,.\,\overrightarrow{DM} = 32$
	find magnitude of a vector	•7 $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$
	find magnitude of a vector	•8 $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$
	evaluate angle BDM	•9 $40{\cdot}3^\circ$ or $0{\cdot}703$ rads
12 P2 Q5	interpret vector	•1 $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ Treat $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ written as $(1,\ 0,\ -1)$ as bad form
	process vector	•2 $\begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$
	use scalar product	•3 $\cos A\hat{B}C = \dfrac{\overrightarrow{BA}\,.\,\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ see Note 1
	find scalar product	•4 3
	find $\lvert\overrightarrow{BA}\rvert$	•5 $\sqrt{2}$
	find expression for $\lvert\overrightarrow{BC}\rvert$	•6 $\sqrt{2^2+(k+3)^2+(-1)^2}$ or equivalent
	complete to result	•7 $\dfrac{3}{\sqrt{2}\sqrt{k^2+6k+14}}$ and $\dfrac{3}{\sqrt{2(k^2+6k+14)}}$ or $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert = \sqrt{2}\times\sqrt{k^2+6k+14}$ and $\dfrac{3}{\sqrt{2(k^2+6k+14)}}$

		Method 1 : Squaring first •[8] $\dfrac{3}{\sqrt{2(k^2+6k+14)}} = \cos 30^\circ$ •[9] $\left(\dfrac{3}{\sqrt{2(k^2+6k+14)}}\right)^2 = \left(\dfrac{\sqrt{3}}{2}\right)^2$ •[10] $k^2+6k+14=6$ or equivalent •[11] $k^2+6k+8=0$ or equivalent •[12] $k=-2$ or -4 = 0 must appear at this stage. Method 2 : Dealing with fractions first •[8] $\dfrac{3}{\sqrt{2(k^2+6k+14)}} = \cos 30^\circ$ •[9] $\sqrt{3}\sqrt{2(k^2+6k+14)}=6$ •[10] $6(k^2+6k+14)=36$ •[11] $k^2+6k+8=0$ or equivalent •[12] $k=-2$ or -4 = 0 must appear at this stage.
13 P1 Q24	use vector approach compare two vectors complete proof state ratio	•[1] $\overrightarrow{AT} = \begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix}$ •[2] $\overrightarrow{TB}$ or $\overrightarrow{AT}$ **and** $\overrightarrow{AT} = \frac{2}{3}\overrightarrow{TB}$ or equivalent •[3] $\overrightarrow{AT}$ and $\overrightarrow{TB}$ are parallel and since there is a common point A, B and T are collinear •[4] 2:3 **stated explicitly** (see Note 4)

	Method 1	**Method 1**
	interpret C	•5 $(c, 0, 0)$
	use vector approach	•6 $\overrightarrow{TC}=\begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$
	know to use scalar product equal to 0	•7 $\overrightarrow{TB}.\overrightarrow{TC}=0$
	start to solve	•8 $15(c-3)+15\times(-2)+6\times(-5)\ \ldots$
	complete	•9 $c=7$
	Method 2	**Method 2**
	interpret C	•5 $(c, 0, 0)$
	use vector approach	•6 $\overrightarrow{TC}=\begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$
	know to use Pythagoras and calculate $\lvert\overrightarrow{TC}\rvert$ or $\lvert\overrightarrow{TB}\rvert$	•7 $\lvert\overrightarrow{TC}\rvert=\sqrt{(c-3)^2+4+25}$
	calculate the other two lengths	•8 $\lvert\overrightarrow{TB}\rvert=\sqrt{486}$ and $\lvert\overrightarrow{BC}\rvert=\sqrt{(c-18)^2+289+121}$
	complete	•9 $c=7$
14 P2 Q4	states coordinates of C	•1 $C(11,12,6)$
	states coordinates of D	•2 $D(8,8,4)$
	finds $\overrightarrow{CB}$	•3 $\begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$
	finds $\overrightarrow{CD}$	•4 $\begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$
	know to use scalar product applied to the correct angle	•5 $\cos B\hat{C}D=\dfrac{\overrightarrow{CB}.\overrightarrow{CD}}{\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{CD}\rvert}$ stated or implied by •9
	find scalar product	•6 40
	find $\lvert\overrightarrow{CB}\rvert$	•7 $\sqrt{80}$
	find $\lvert\overrightarrow{CD}\rvert$	•8 $\sqrt{29}$
	find angle	•9 $33\cdot9^{\circ}$

PRE 2000 – ANSWERS - **Components, Magnitude and Unit Vectors**

1	•[1] pathway for $\overrightarrow{CV}$: $\overrightarrow{CV}=\overrightarrow{CA}+\overrightarrow{AV}$ •[2] *e.g.* $\overrightarrow{CB}=2i-10j+2k$ ***or*** $\overrightarrow{BA}=-8i-2j-2k$ ***or*** $\overrightarrow{AC}=6i+12j$ •[3] $\begin{pmatrix}-5\\-5\\7\end{pmatrix}$
2	•[1] $q-p=8i-4j+k$ ***or*** $p=\begin{pmatrix}-1\\3\\4\end{pmatrix}$, $q=\begin{pmatrix}7\\-1\\5\end{pmatrix}$ •[2] $\overrightarrow{PQ}=\begin{pmatrix}8\\-4\\1\end{pmatrix}$ •[3] 9
3	•[1] $\sqrt{2^2+(-3)^2+(\sqrt{3})^2}$ stated or implied by •[2] •[2] 4
4	•[1] $\begin{pmatrix}2\\1\\-2\end{pmatrix}$ •[2] $\sqrt{(-3+1)^2 + (2-3)^2 + (4-2)^2}$ •[3] 3
5	•[1] $\overrightarrow{BC}=\begin{pmatrix}4\\2\\-3\end{pmatrix}$ •[2] $\sqrt{29}$
6	•[1] $6i-7j+6k$ •[2] $\sqrt{6^2+(-7)^2+6^2}$ •[3] 11
7	•[1] $(x,0,0)$ or equiv. •[2] $(x-4)^2+4+36=49$ or equiv. •[3] $x=1,\ 7$ ***OR*** •[1] $PQ=\sqrt{40}$ •[2] $d=3$ •[3] (1,0,0), (7,0,0) ***OR*** •[1] $d^2=7^2-6^2-2^2$ •[2] $d=3$ •[3] (1,0,0), (7,0,0)
8	•[1] $p=\begin{pmatrix}1\\1\\-1\end{pmatrix}$, $q=\begin{pmatrix}1\\0\\4\end{pmatrix}$, $r=\begin{pmatrix}4\\-3\\0\end{pmatrix}$ *s/i by* •[2] •[2] $\begin{pmatrix}8\\-5\\-5\end{pmatrix}$ •[3] 1 •[4] 5
9	•[1] $\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AD}$ or equivalent, stated or implied by •[3] •[2] $\overrightarrow{BC}=\begin{pmatrix}-13\\3\\-1\end{pmatrix}$ ***or*** $\overrightarrow{CB}$ or $\overrightarrow{AB}$ or $\overrightarrow{BA}$ •[3] D= (−11, 2, 3) •[1] $\overrightarrow{OD}=\overrightarrow{OM}+\overrightarrow{MD}$, M is midpoint of AC •[2] $\overrightarrow{BM}=\begin{pmatrix}-9\\\frac{1}{2}\\0\end{pmatrix}$ •[3] $D=(-11,\ 2,\ 3)$

10	•1 $\overrightarrow{QP}=\begin{pmatrix}-3\\5\\5\end{pmatrix}$ •2 $R=(3,1,1)$ and $\overrightarrow{RS}=\begin{pmatrix}-3\\5\\5\end{pmatrix}$ stated or implied by •3 •3 $S=(0,6,6)$

ANSWERS PRE 2000 - **Collinearity**

1	•1 $\overrightarrow{AB}=\begin{pmatrix}1\\-4\\2\end{pmatrix}$ •2 $\overrightarrow{BC}=\begin{pmatrix}2\\-8\\4\end{pmatrix}$ **AND** $\overrightarrow{BC}=2\times\overrightarrow{AB}$ •3 $\overrightarrow{AB}\,\|\,\|\overrightarrow{BC}$ & B is common hence A, B, C collinear
2	•1 $\overrightarrow{AB}=\begin{pmatrix}4\\2\\-2\end{pmatrix}$ *or* $\overrightarrow{AC}=\begin{pmatrix}10\\5\\-5\end{pmatrix}$ *or* $\overrightarrow{BC}=\begin{pmatrix}6\\3\\-3\end{pmatrix}$ •2 $\overrightarrow{AB}=2\begin{pmatrix}2\\1\\-1\end{pmatrix}$ *and* $\overrightarrow{BC}=3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or equivalent •3 $AB\,\|\,\|BC$ **and** B **is** point in common •4 2:3 (*or* equivalent e.g. $1:1\frac{1}{2}$)
3	•1 $\overrightarrow{ST}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ or equivalent **and** $\overrightarrow{RS}=\begin{pmatrix}3\\3\\6\end{pmatrix}$ or equivalent •2 $\overrightarrow{RS}=3\overrightarrow{ST}$ or equiv. •3 RS // ST **and** S is common.
4	•1 $\overrightarrow{LM}=\begin{pmatrix}12\\-8\\4\end{pmatrix}$ or equivalent combinations for (a) •2 $\overrightarrow{MN}=\begin{pmatrix}3\\-2\\1\end{pmatrix}$ •3 $\overrightarrow{LM}=4\overrightarrow{MN}$ •4 vectors are parallel and have common point so L, M, N are collinear •5 4:1
5	•1 $\overrightarrow{PQ}=\begin{pmatrix}2\\2\\-2\end{pmatrix}$ •2 $\overrightarrow{QR}=\begin{pmatrix}1\\1\\-1\end{pmatrix}=\frac{1}{2}\overrightarrow{PQ}$ **or equivalent** •3 vectors parallel and have pt in common so pts collinear •4 $PQ:QR=2:1$

ANSWERS PRE 2000 – **Section Formula**

1	•1 $\overrightarrow{AB}=\begin{pmatrix}6\\3\\-6\end{pmatrix}$ •2 $\overrightarrow{AC}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ •3 $C=(5,0,0)$ •4 $D=(7,1,-2)$

2	•[1] $\overrightarrow{AB}=\begin{pmatrix}4\\-2\\-6\end{pmatrix}$ •[2] $\overrightarrow{BC}=\overrightarrow{AB}$ •[3] $(5,0,-5)$
3	(a) •[1] $(6,1,1)$ (b) •[2] e.g. $\overrightarrow{CM}=\begin{pmatrix}6\\-3\\-3\end{pmatrix}$ •[3] $\overrightarrow{CT}=\begin{pmatrix}4\\-2\\-2\end{pmatrix}$ •[4] $T=(4,2,2)$ (c) •[5] e.g. $\overrightarrow{BT}=\begin{pmatrix}-4\\-1\\3\end{pmatrix}$ •[6] $\overrightarrow{TD}=\begin{pmatrix}-8\\-2\\6\end{pmatrix}=2\times\overrightarrow{BT}$ •[7] TD is parallel to BT, T is common point so B, T, D collinear •[8] BT:TD = 1:2

PRE 2000 – ANSWERS – **The Dot Product**

1	•[1] $p=\begin{pmatrix}1\\1\\-1\end{pmatrix},\ q=\begin{pmatrix}1\\0\\4\end{pmatrix},\ r=\begin{pmatrix}4\\-3\\0\end{pmatrix}$ s/i by •[2] •[2] $\begin{pmatrix}8\\-5\\-5\end{pmatrix}$ •[3] 1 •[4] 5
2	•[1] $\mathbf{u}+\mathbf{v}=\begin{pmatrix}-2\\8\\2\end{pmatrix}$ and $\mathbf{u}-\mathbf{v}=\begin{pmatrix}-4\\-2\\4\end{pmatrix}$ •[2] $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v})=8-16+8$ •[3] $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v})=0$ so $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are perpendicular
3	•[1] *strat:* $\boldsymbol{a}.\boldsymbol{b}=\ldots\ldots\ldots$ •[2] $\boldsymbol{a}.\boldsymbol{b}=0\Rightarrow$ perpendicularity explicitly stated •[3] $\begin{pmatrix}2\\3\\-1\end{pmatrix}.\begin{pmatrix}3\\-1\\3\end{pmatrix}=6-3-3=0$
4	•[1] $\overrightarrow{AB}=\begin{pmatrix}-6\\-8\\2\end{pmatrix},\overrightarrow{BC}=\begin{pmatrix}-6\\4\\-2\end{pmatrix},\overrightarrow{AC}=\begin{pmatrix}-12\\-4\\0\end{pmatrix}$ •[2] $\lvert\overrightarrow{AC}\rvert$ is longest so $\overrightarrow{AB}.\overrightarrow{CB}=-36+32+4=0$ •[3] $A\hat{B}C=90°$

5	•1 $\begin{pmatrix}1\\2\\-1\end{pmatrix}.\begin{pmatrix}-4\\3\\k-1\end{pmatrix}=0$ •2 $1\times-4+2\times3+-1(k-1)$ •3 3
6	•1 $\begin{pmatrix}a\\b\\1\end{pmatrix}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=a-b+1$ *or* $\begin{pmatrix}a\\b\\1\end{pmatrix}.\begin{pmatrix}-2\\1\\1\end{pmatrix}=-2a+b+1$ •2 $a-b+1=0$ *or* $-2a+b+1=0$ •3 $a=2$ *and* $b=3$
7	•1 equate scalar product to zero •2 state value of t
	•1 $-24+2t+6=0$ •2 $t=9$

ANSWERS PRE 2000 – **Calculating Angles**

1	**(a)** •1 $\overrightarrow{AB}=\begin{pmatrix}1\\7\\2\end{pmatrix}$ •2 $\overrightarrow{AC}=\begin{pmatrix}4\\7\\-5\end{pmatrix}$ **(b)** •3 $\cos B\hat{A}C=\dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\,\lvert\overrightarrow{AC}\rvert}$ *stated or implied by responses to •4 to •7* •4 $\overrightarrow{AB}.\overrightarrow{AC}=4+49-10$ •5 $\overrightarrow{AB}=\sqrt{54}$ •6 $\overrightarrow{AC}=\sqrt{90}$ •7 $B\hat{A}C=51\cdot9^\circ$ **(c)** •8 **identify** 2 sides and included angle *e.g.* $\sqrt{54},\ \sqrt{90},\ B\hat{A}C$ •9 $27\cdot4$
2	**(a)** •1 One of B, C or D •2 Remaining two of B, C and D •3 B(6,4,2), C(4,3,4), D(6,2,2) **(b)** •4 $\left(\frac{2+6}{2},\frac{4+2}{2},\frac{6+2}{2}\right)$ **(c)** •5 $\cos A\hat{O}B=\dfrac{\overrightarrow{OA}.\overrightarrow{OB}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert}$ *or* $\dfrac{OA^2+OB^2-AB^2}{2\times OA\times OB}$ or equivalents •6 $\overrightarrow{OA}.\overrightarrow{OB}=40$ *or* $AB^2=32$ •7 $OA=\sqrt{56}=OB$ •8 44° **(d)** •9 strategy: **e.g.** use isosceles Δ •10 68°

3	**(a)** •1 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ •2 $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$ •3 $R = (7,-1,6)$ **(b)** •4 $\overrightarrow{SP}.\overrightarrow{SR} = \lvert SP\rvert\lvert SR\rvert\cos P\hat{S}R$ •5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$ •7 $\lvert SP\rvert = \sqrt{11}$ •8 $\lvert SR\rvert = \sqrt{91}$ •9 $\overrightarrow{SP}.\overrightarrow{SR} = 3$ •10 $P\hat{S}R = 84\cdot 6^\circ$
4	**(a)** •1 $PQ = \sqrt{8},\ RQ = \sqrt{8},$ •2 Use s.p.: $\overrightarrow{PQ}.\overrightarrow{RQ} = \lvert\overrightarrow{PQ}\rvert.\lvert\overrightarrow{RQ}\rvert\cos\theta$ •3 $\begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}.\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} = 4$ •4 60° **(b)** •5 $M = (2,3,2)$ •6 $\overrightarrow{PT} = \frac{2}{3}\overrightarrow{PM}$ or equivalent •7 $\overrightarrow{PT} = \frac{2}{3}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ or equiv. •8 $T = \left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$ •9 $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ stated or implied •10 $PT = 2\sqrt{\frac{2}{3}}, QT = 2\sqrt{\frac{2}{3}}, RT = 2\sqrt{\frac{2}{3}}$ or equivalent **(c)** •11 $PA = QA = RA = \sqrt{3}$ •12 A is in a different plane
5	**(a)** •1 $A(1,0,0)$ •2 $B(3,2,0)$ •3 $C(3,0,-2)$ **(b)** •4 strategy for area of triangle and attempt to calculate parts •5 60° or altitude $= \sqrt{6}$ •6 side $= 2\sqrt{2}$ •7 using chosen formula correctly **(c)** •8 54 unit^2 for cube •9 know how to calculate s.a of crystal •10 area of 1 pentagonal face = 7 unit^2 •11 51.5 unit^2 for crystal $(48 + 2\sqrt{3})$ •12 strategy for finding % decrease

6

(a) •1 Strategy: use vectors or 3-D distance formula

•2 $\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(b) •4 $|\overrightarrow{MR}| = \sqrt{115.25}$ or equivalent

•5 answer

(c) •6 know to use a scalar product

•7 $\overrightarrow{TC}.\overrightarrow{BR} = 0$

•8 communication: 0 ⇔ perpendicularity

(d) •9 Strategy: know to use

$\cos T\hat{C}R = \dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|TC||RC|}$ or equiv.

•10 $\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\overrightarrow{TC}.\overrightarrow{RC} = 82$

•13 36.7°

7

(a) •1 M = (6, 18, 12)

•2 *e.g.* $\overrightarrow{BG} = \frac{2}{3}\begin{pmatrix} -21 \\ 15 \\ 12 \end{pmatrix}$

•3 G = (13, 13, 8)

(b) •4 $\cos A\hat{O}G = \dfrac{\overrightarrow{OA}.\overrightarrow{OG}}{|\overrightarrow{OA}|\,|\overrightarrow{OG}|}$

•5 $\overrightarrow{OA} = \begin{pmatrix} 9 \\ 9 \\ 24 \end{pmatrix}$ and $\overrightarrow{OG} = \begin{pmatrix} 13 \\ 13 \\ 8 \end{pmatrix}$

•6 $\overrightarrow{OA}.\overrightarrow{OG} = 426$

•7 $|\overrightarrow{OA}| = \sqrt{738}$ and $|\overrightarrow{OG}| = \sqrt{402}$

•8 38.5°

8

(a) •1 obtaining for example $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

•2 $\overrightarrow{AK} = \begin{pmatrix} -5 \\ 5 \\ 11 \end{pmatrix}$

(b) •3 obtaining for example $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

•4 $\overrightarrow{AL} = \begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix}$

(c) •5 *strategy e.g.* $\cos K\hat{A}L = \frac{\overrightarrow{AK}.\overrightarrow{AL}}{|AK| \times |AL|}$

•6 109

•7 $\sqrt{171}$

•8 $\sqrt{101}$

•9 $\hat{A} = 34.0$

OR

•5 *strategy e.g.* $\cos K\hat{A}L = \frac{AK^2 + AL^2 - KL^2}{2AK \times AL}$

•6 $\sqrt{54}$

•7 $\sqrt{171}$

•8 $\sqrt{101}$

•9 $\hat{A} = 34.0$

9	**(a)** •[1] $Q=(2,2,9)$ •[2] $R=(21,3,12)$ **(b)** •[3] $\cos\theta=\frac{a.b}{\lvert a\rvert\lvert b\rvert}$ *with some subsequent use* *eg* $\cos Q\hat{P}R=\frac{\overrightarrow{PQ}.\overrightarrow{PR}}{\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{PR}\rvert}$ •[4] $\overrightarrow{PQ}=\begin{pmatrix}-10\\2\\9\end{pmatrix}$ •[5] $\overrightarrow{PR}=\begin{pmatrix}9\\3\\12\end{pmatrix}$ •[6] $\lvert\overrightarrow{PQ}\rvert=\sqrt{185}$ •[7] $\lvert\overrightarrow{PR}\rvert=\sqrt{234}$ •[8] $\overrightarrow{PQ}.\overrightarrow{PR}=24$ •[9] $Q\hat{P}R=83\cdot4^\circ$
10	(a) •[1] $\overrightarrow{VF}=\begin{pmatrix}1\\1\\-10\end{pmatrix}$ •[2] $E=(2,0,-7)$ •[3] $\overrightarrow{VE}=\begin{pmatrix}1\\-1\\-10\end{pmatrix}$ •[4] $\cos E\hat{V}F=\frac{\overrightarrow{VE}.\overrightarrow{VF}}{\lvert\overrightarrow{VE}\rvert\,\lvert\overrightarrow{VF}\rvert}$ This may appear as $\frac{100}{102}$ **after the completion of •[5] and •[6].** •[5] $\overrightarrow{VE}.\overrightarrow{VF}=100$ •[6] $\lvert\overrightarrow{VE}\rvert\,\lvert\overrightarrow{VF}\rvert=102$ •[7] 11.4° (b) •[8] $\frac{1}{2}VE\times VF\sin E\hat{V}F$ •[9] $\frac{1}{2}\times102\times\sin11.4^\circ$ •[10] 10.02

ANSWERS PRE 2000 – **Algebraic Properties of the Dot Product**

1	•[1] $\boldsymbol{a.a+a.b}$ •[2] $2\times3\times\cos60^\circ$ •[3] 4
2	•[1] $\boldsymbol{b.a+b.b+b.c}$ •[2] $\boldsymbol{b.a}=0$ •[3] $\boldsymbol{b.b}=4$ •[4] $\lvert\boldsymbol{c}\rvert=2\sqrt{2}$ •[5] $\boldsymbol{b.c}=4$

3	•[1] $\boldsymbol{p}.\boldsymbol{q}+\boldsymbol{p}.\boldsymbol{r}$ •[2] $V\hat{A}D = 60^\circ$ or equiv. •[3] $\lvert\boldsymbol{p}\rvert\lvert\boldsymbol{q}\rvert\cos V\hat{A}D + \lvert\boldsymbol{p}\rvert\lvert\boldsymbol{r}\rvert\cos V\hat{A}B$ •[4] 9 •[1] $\boldsymbol{r}=\begin{pmatrix}0\\3\\0\end{pmatrix}, \boldsymbol{q}=\begin{pmatrix}-3\\0\\0\end{pmatrix}$ •[2] $\boldsymbol{p}=\begin{pmatrix}-\frac{3}{2}\\\frac{3}{2}\\\frac{3}{\sqrt{2}}\end{pmatrix}$ •[3] $\left(-\frac{3}{2}\right)\times(-3)+\left(\frac{3}{2}\right)\times 3+\frac{3}{\sqrt{2}}\times 0$ •[4] 9
4	•[1] $\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}$ •[2] $\boldsymbol{a}.\boldsymbol{b} = 2\times 2\times \frac{1}{2}$ •[3] $\boldsymbol{a}.\boldsymbol{c} = 2\times 2\times -\frac{1}{2}$ •[4] 0 **and** $\boldsymbol{a}$ is perpendicular to $(\boldsymbol{b}+\boldsymbol{c})$
5	•[1] $a=\begin{pmatrix}2\\0\\-1\end{pmatrix}, b=\begin{pmatrix}1\\2\\1\end{pmatrix}, c=\begin{pmatrix}0\\-1\\1\end{pmatrix}$ •[2] $\boldsymbol{a}.\boldsymbol{b}=1$ •[3] $\boldsymbol{a}.\boldsymbol{c}=-1$ •[4] $\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}=\boldsymbol{a}.(\mathbf{b}+\mathbf{c})$ •[5] $\boldsymbol{a}\perp \mathbf{b}+\mathbf{c}$
6	•[1] $\boldsymbol{a}.\boldsymbol{a}+\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}$ •[2] $a.a = \lvert a\rvert\lvert a\rvert\cos 0$ •[3] $a.b = \lvert a\rvert\lvert b\rvert\cos 60$ •[4] $a.c = \lvert a\rvert\lvert c\rvert\cos 120$ •[5] 4
7	•[1] $a.a=9$ **and** $b.b=8$ •[2] $a.b=6$ •[3] $(2a+3b).(2a+3b)$ •[4] $4a.a+9b.b+12a.b$ •[5] 180 •[6] $\sqrt{180}$